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Deccan Education Society's

Kirti M. Doongursee College of Arts, Science and Commerce (AUTONOMOUS)

Affiliated to

UNIVERSITY OF MUMBAI

Syllabus for
Program: Bachelor of Science
Course: F.Y.B.SC.
Subject: Mathematics

Choice Based Credit System (CBCS)
with effect from
Academic Year 2022-2023

About the course:

Kirti M. Doongursee College of Arts, Science and Commerce (AUTONOMOUS) has brought into force the revised syllabi as per the Choice Based Credit System (CBCS) for the First year B. Sc. Programme in Mathematics from the academic year 2022-2023.

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects like Physics, Statistics and Computer Sciences, the teachers of Mathematics from the department with consent of Board of Studies in Mathematics has prepared the syllabus of F.Y.B. Sc. Mathematics. The present syllabi of F. Y. B. Sc. for Semester I and Semester II has been designed as per U. G. C. Model curriculum so that the students learn Mathematics needed for these branches, learn basic concepts of Mathematics and are exposed to rigorous methods gently and slowly. The syllabi of F. Y. B. Sc. would consist of two semesters and each semester would comprise of two courses for F. Y. B. Sc. Mathematics. Course I is 'Calculus I and Calculus II'. Calculus is applied and needed in every conceivable branch of science. Course II, 'Algebra I and Discrete Mathematics' develops mathematical reasoning and logical thinking and has applications in science and technology.

Course Objectives:

- Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling, solving and interpreting.
- Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science.
- Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment.
- A student should get adequate exposure to global and local concerns that

explore them many aspects of Mathematical Sciences

Learning Outcomes:

1. Calculus (Sem I & II): This course gives introduction to basic concepts of Analysis with rigor and prepares students to study further courses in Analysis. Formal proofs are given lot of emphasis in this course which also enhances understanding of the subject of Mathematics as a whole. The portion on first order, first degree differential equations prepares the learner to get solutions of so many kinds of problems in all subjects of Science and also prepares learner for further studies of differential equations and related fields.
2. Algebra I (Sem I) & Discrete Mathematics (Sem II): This course gives expositions to number systems (Natural Numbers & Integers), like divisibility and prime numbers and their properties. These topics later find use in advanced subjects like cryptography and its uses in cyber security and such related fields.

F. Y. B. Sc. (CBCS) SEMESTER – I

Course Code	Course Title	Credits	Lectures /Week
KUSMT22101	Paper – I Calculus – I	2	3
KUSMT22102	Paper – II Algebra – I	2	3

CALCULUS – I

Unit	Topics	No of Lectures
I	Real Number System	15
II	Sequences in R	15
III	First Order First Degree Differential Equations	15
	ALGEBRA I	
I	Integers and Divisibility	15
II	Functions, Relations and Binary Operations	15
III	Polynomials	15

SEMESTER I

KUSMT22101: CALCULUS I

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit 1 : Real Number System (15 Lectures)

(i) Real number system R and order properties of R, absolute value | |, and its properties.

(ii) AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighborhoods, interior points, limit point, Hausdorff property.

(iii) Bounded sets, statements of I. u. b. axiom and its consequences, supremum and infimum, maximum and minimum, Archimedean property and its applications, density of rational numbers.

Unit II: Sequences in R (15 lectures)

(i) Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

(ii) Convergence of standard sequences like $\frac{1}{1+na}$, $\forall\, a > 0$, (b^n), $0 < b < 1$, $\left(c^{\frac{1}{n}}\right)$, $c > 0$, $\left(n^{\frac{1}{n}}\right)$, $n \in N$, $n > 1$.

(iii) Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone con- vergence theorem and consequences as convergence of $\left(\left(1 + \frac{1}{n}\right)^n\right)$.

(iv) Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequence is a Cauchy sequence and converse.

Unit III: First order First degree Differential equations (15 Lectures)

Review of Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE. Solution of homogeneous and non- homogeneous differential equations of first order and first degree. Notion of partial derivatives.

(1) Exact Equations: General solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations: Rules for finding integrating factors (without proof) for non-exact equations, such as :

(i) $\frac{1}{Mx+Ny}$ is an I.F. and $Mx + Ny \neq 0$, and $Mdx + Ndy = 0$ is a homogeneous differential equation.

(ii) $\frac{1}{Mx - Ny}$ is an I.F. and $Mx - Ny \neq 0$, and $Mdx + Ndy = 0$ is a differential equation of the form $f_1(xy)ydx + f_2(xy)xdy = 0$.

(iii) $e^{\int f(x)dx}$ is an I.F and N $\neq 0$, $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function of x alone, say *f(x)*.

(iv) $e^{\int g(y)dy}$ is an I.F and N $\neq 0$, $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$ is a function of y alone, say *g(y)*.

(v) Linear and reducible linear equations of first order, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

(2) Reduction of order :

(i) If the differential equation does not contain only the original function y, that is, equations of type $F(x, y', y'') = 0$.

(ii) If the differential equation does not contain the independent variable x that is, equations of type $F(y, y', y'') = 0$.

Textbooks:

1. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
2. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
3. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
4. G. F. Simmons, Differential Equations with Applications and Historical Notes, McGraw Hill, 1972.
5. E. A. Coddington, An Introduction to Ordinary Differential

Equations, Prentice Hall, 1961.

Additional References:

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit Kumar and S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing Company, 1994.
5. D. A. Murray, Introductory Course in Differential Equations, Longmans, Green and Co., 1897.
6. A. R. Forsyth, A Treatise on Differential Equations, Macmillan and Co.,1956.

ALGEBRA – I
KUSMT22202

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations ${}^{n}P_{r}$ and Combinations ${}^{n}C_{r}$.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I : Integers & Divisibility (15 Lectures)

(1) Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of Well-Ordering Principle.

(2) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two non-zero integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of two non-zero integers a and b and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in Z$, Euclidean algorithm.

(3) Primes, Euclid's lemma, Fundamental Theorem of arithmetic, the set of primes is infinite, there are arbitrarily large gaps between primes, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.

(4) Congruence, definition and elementary properties, Results about linear congruence equations. Examples.

Unit II : Functions, Relations and Binary Operations
(15 Lectures)

(1) Definition of relation and function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely, examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.

(2) Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

(3) Congruence is an equivalence relation on Z, Residue classes and partition of Z, Addition modulo n, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)

(1) Definition of a polynomial, polynomials over F where $F = Q, Q$ or C, Algebra of polynomials, degree of polynomial, basic properties.

(2) Division algorithm in $F[X]$ (without proof), and g.c.d of two polynomials and its basic properties, Euclidean algorithm (proof of the above results may be given only in the case of $Q[X]$ with a remark that the results as well as the proofs remain valid in the case of $R[X]$ or $C[X]$).

(3) Roots of a polynomial, relation between roots and coefficients, multiplicity of a root.

Elementary consequences such as the following.

(i) Remainder theorem, Factor theorem.

(ii) A polynomial of degree n has at most n roots.

(iii) Complex and non-real roots of a polynomials in R[X] occur in conjugate pairs. (Emphasis on examples and problems in polynomials with real coefficients).

(4) Necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients (viz. p divides the constant coefficient and q divides the leading coefficient), corollary for monic polynomials (viz. a rational root of monic polynomial with integer coefficients is necessarily an integer). Simple consequence such as the irrationality is necessarily of $\sqrt{p}$ for any prime number p. Irreducible polynomials in Q[x], Unique Factorization Theorem.

Examples.

Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.

2.2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.

F. Y. B. Sc. (CBCS) SEMESTER – II

Course Code	Course Title	Credits	Lectures /Week
KUSMT22201	**Paper – I Calculus – II**	**2**	**3**
KUSMT22202	**Paper – II Discrete Mathematics**	**2**	**3**

CALCULUS – II

Unit	Topics	No of Lectures
I	Limits and Continuity	**15**
II	Differentiability of functions	**15**
III	Applications of Differentiability	**15**
	DISCRETE MATHEMATICS	
I	Preliminary Counting	**15**
II	Advanced Counting	**15**
III	Permutations and Recurrence Relation	**15**

SEMESTER II

KUSMT22201: CALCULUS – II

{Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function. Graphs of some standard functions such as $|x|$, e^x, $\log x$, $ax^2 + bx + c$, $\frac{1}{x}$, x^n, $n \geq 3$, $\sin x$, $\cos x$, $\tan x$, $\sin^{-1}x$, $\sin\left(\frac{1}{x}\right)$, $x^2\sin\left(\frac{1}{x}\right)$ over suitable intervals of R: No direct questions to be added.}

(1) $\epsilon - \delta$ definition of limit of a function, uniqueness of limit if it exists, algebra of limits, limits of composite functions, Sandwich theorem, left hand limit $f(x)$, right hand limit $f(x)$, non-existence of limits, $f(x)$, $f(x)$, $f(x) = \pm\infty$.

(2) Continuous functions: Continuity of a real valued function at a point and on a set using $\epsilon - \delta$ definition, examples, Continuity of a real valued function at end points of domain using $\epsilon - \delta$ definition, f is continuous at a if and only if $f(x)$ exists and equals to f(a), Sequential continuity, Algebra of continuous functions,

discontinuous functions, examples of removable and essential discontinuity.

(3) Intermediate Value theorem and its applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

Unit-II: Differentiability of functions (15 Lectures)

(1) Differentiation of real valued function of one variable: Definition of differentiability of a function at a point of an open interval, examples of differentiable and non-differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

(2) Chain rule, Higher order derivatives, Leibniz rule, Derivative of inverse functions, Implicit differentiation (only examples)

Unit-III: Applications of differentiability (15 Lectures)

(1) Rolle's Theorem, Lagrange's and Cauchy's Mean Value Theorems, applications and examples, Monotone increasing and decreasing functions, examples.

(2) L-Hospital rule (without proof), examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

(3) Definition of critical point, local maximum/minimum, necessary condition, stationary points, second derivative test, examples, concave/convex functions, point of inflection.

(4) Sketching of graphs of functions using properties

Reference books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.

3. T. M. Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Lt 11

4. Sudhir Ghorpade and Balmohan Limaye, A course in Calculus and Real Analysis, Springer International Ltd, 2000.

Additional Reference:

1. Richard Courant and Fritz John, A Introduction to Calculus and Analysis,

Volume-I, Springer.

2. Ajit Kumar and S. Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.

3. K. G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

4. G. B. Thomas, Calculus, 12th Edition 2009

KUSMT22202: DISCRETE MATHEMATICS

Unit I: Preliminary Counting (15 Lectures)

(1) Finite and infinite sets, countable and uncountable sets examples such as N, Z, $N \times N$, Q, $(0, 1)$, R.

(2) Addition and multiplication Principle, counting sets of pairs, two ways counting.

(3) Stirling numbers of second kind. Simple recursion formulae satisfied by $S(n, k)$ for $k = 1, 2, \cdots, n-1, n$.

(4) Pigeonhole principle simple and strong form and examples, its applications to geometry.

Unit II: Advanced Counting (15 Lectures)

(1) Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.

(2) Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such a s the following with emphasis on combinatorial proofs.

- $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

- $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$

- $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

- $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

(3) Non-negative integer solutions of equation $x_1 + x_2 + \cdots + x_k = n$.

(4) Principal of inclusion and exclusion, its applications, derangements, explicit formula for d_n, deriving formula for Euler's function $\varphi(n)$.

Unit III: Permutations and Recurrence relation (15 lectures)

(1) Permutation of objects, S_n, composition of permutations, results such as every permutation is a product of disjoint cycles, every cycle is a product of transpositions, signature of a permutation, even and odd permutations, cardinality of S_n, A_n.

(2) Recurrence Relations, definition of homogeneous, non-homogeneous, linear, non-linear recurrence relation, obtaining recurrence relations of Tower of Hanoi, Fibonacci sequence, etc. in counting problems, solving homogeneous as well as non-homogeneous recurrence relations by using iterative methods, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended Books:

1. Norman Biggs, Discrete Mathematics, Oxford University Press.
2. Richard Brualdi, Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy, Combinatorics-Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series, Discrete mathematics,
6. Allen Tucker, Applied Combinatorics, John Wiley and Sons.
7. Sharad Sane, Combinatorial Techniques, Springer

PRACTICALS FOR F. Y. B. Sc.

SEMESTER – I

Course Code	Course Title	Credits	Lectures/ Week
KUSMTP22101	Practicals for **KUSMT22101 + KUSMT22102**	1	1
KUSMTP22101 **Paper 1**			
1	Algebraic and Order Properties of Real Numbers and Inequalities		
2	Hausdorff Property and LUB Axiom of R, Archimedian Property.		
3	Convergence and divergence of sequences, bounded sequences, Sandwich Theorem.		
4	Cauchy sequences, monotonic sequences, non-monotonic sequences.		
5	Solving exact and non-exact, linear, reducible to linear differential equations.		
6	Reduction of order of Differential Equations, Applications of Differential Equations.		
7	Miscellaneous Theoretical Questions based on full paper.		
Paper 2			
1	Mathematical induction ,Division Algorithm, Euclidean algorithm in Z, Examples on expressing the g. c. d. of two non-zero integers a and b as $ma + nb$ for some $m, n \in$ Z,		
2	Primes and the Fundamental theorem of Arithmetic, Euclid's lemma, there exists infinitely many primes of the form $4n - 1$ or of the form $6n - 1$.		
3	Functions, Bijective and Invertible functions, Compositions of functions.		

4	Binary Operation, Equivalence Relations, Partition and Equivalence classes
5	Polynomial (I)
6	Polynomial (II)
7	Miscellaneous Theoretical Questions based on full paper.

PRACTICALS FOR SEMESTER – II

Course Code	**Course Title**	**Credits**	**Lectures/ Week**
KUSMTP22201	**Practicals for KUSMT22201 + KUSMT22202**	**1**	**1**
KUSMTP22201 Paper 1			
1	Limit of a function and Sandwich theorem, Continuous and discontinuous function		
2	Algebra of limits and continuous functions, Intermediate Value theorem, Bolzano-Weierstrass theorem		
3	Properties of differentiable functions, derivatives of inverse functions and implicit functions		
4	Higher order derivatives, Leibnitz Rule		
5	Mean value theorems and its applications, L'Hospital's Rule, Increasing and Decreas- ing functions		
6	Extreme values, Taylor's Theorem and Curve Sketching		
7	Miscellaneous Theoretical Questions based on full paper		
Paper 2			
1	Counting principles, Two way counting		
2	Stirling numbers of second kind, Pigeon hole principle		
3	Multinomial theorem, identities, permutation and combination of multi-set.		
4	Inclusion-Exclusion principle. Euler phi function		
5	Composition of permutations, signature of permutation, inverse of permutation		

6	Recurrence relation.
7	Miscellaneous Theoretical Questions based on full paper

Evaluation Scheme for First Year (UG) under AUTONOMY

Scheme of Examination (60:40)

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 40 percent marks in the first part and by conducting the Semester End Examinations with 60 percent marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below: -

I. Internal Evaluation for Theory Courses – 40 Marks

(i) Continuous Internal Assessment 1 Assignment/Tutorial– 20 Marks

(ii) Continuous Internal Assessment 2 – 20 Marks

5 questions of Fill in the blanks (5 marks) + 5 questions True or False (5 marks) + 5 simple questions (10 marks)

II. External Examination for Theory Courses – 60 Marks

Duration: 2 Hours

Theory question paper pattern:

All questions are compulsory.

Question	**Based on**	**Options**	**Marks**
Q.1	12 MCQs	*No option*	12
Q.2	Unit I	*Any 3 out of 4*	12
Q.3	Unit II	*Any 3 out of 4*	12
Q.4	Unit III	*Any 3 out of 4*	12
Q.5	Units I, II, III	*Any 3 out of 6*	12

All questions shall be compulsory with internal choice within the questions.

Each Question may be sub-divided into sub questions as a, b, c, d, etc.

& the allocation of Marks depends on the weightage of the topic.

III. Practical Examination

Each core subject carries 50 Marks

30 marks External – there will be totally 3 questions – 1 question per unit. Each question will have 3 sub–questions a, b, c of which students solve any 2 (8 marks for each question and 6 marks viva)

All questions are compulsory.

Question	Based on	Options	Marks
Q.1	Unit I	*Any 2 out of 3*	8
Q.2	Unit II	*Any 2 out of 3*	8
Q.3	Unit III	*Any 2 out of 3*	8
Viva	All units	--------------------	6

20 marks Internals- Two questions - each with 2 sub–questions to be solved from 3 sub–questions (8 marks each)

Question	Based on	Options	Marks
Q.1	Unit I, Unit II, Unit III	*Any 2 out of 3*	8
Q.2	Unit I, Unit II, Unit III	*Any 2 out of 3*	8
Journals	No of experiments completed		4

Duration: 2 Hours for each practical course.

Minimum 80% practical from each core subjects are required to be completed.

Certified Journal is compulsory for appearing at the time of Practical Exam