(UNIVERSITY OF MUMBAI)

Syllabus for: F.Y.B.Sc./F.Y.B.A.

Program: B.Sc./B/A.
Course: Mathematics
Choice based Credit System (CBCS)
with effect from the
academic year 2018-19

SEMESTER I

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 101, UAMT 101	I	Real Number System	2	3
	II	Sequences		
	III	Limits and Continuity		
ALGEBRA I				
USMT 102	I	Integers and Divisibility	2	3
	II	Functions and equivalence Relation		
	III	Polynomials		
PRACTICALS				
USMTP01	-	Practicals based on USMT101, USMT102	2	2

SEMESTER II

CALCULUS I				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 201, UAMT 201	I	Infinite Series	2	3
	II	Continuous functions and Differentiation		
	III	Applications of Differentiability		
ALGEBRA II				
USMT 102	I	System of Linear Equations and Matrices	2	3
	II	Vector Spaces		
	III	Basis & Linear Transformation		
PRACTICALS				
USMTP02	-	Practicals based on USMT201, USMT202	2	2

Teaching Pattern for Semester I

[1.] Three lectures per week per course. Each lecture is of 48 minutes duration.

[2.] One Practical (2L) per week per batch for courses USMT101, USMT 102 combined (the batches to be formed as prescribed by the University. Each practical session is of 48 minutes duration.)

[3.] One Tutorial per week per batch for the course UAMT101 (the batches to be formed as prescribed by the University). Each tutorial session is of 48 minutes.

Teaching Pattern for Semester II

[1.] Three lectures per week per course. Each lecture is of 48 minutes duration.

[2.] One Practical (2L) per week per batch for courses USMT201, USMT 202 combined (the batches to be formed as prescribed by the University. Each practical session is of 48 minutes duration.)

[3.] One Tutorial per week per batch for the course UAMT101 (the batches to be formed as prescribed by the University). Each tutorial session is of 48 minutes.

F.Y.B.Sc. / F.Y.B.A. Mathematics

SEMESTER I

USMT 101, UAMT 101: CALCULUS I

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit 1 : Real Number System (15 Lectures)
Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, absolute value $|\ |$ and its properties.

AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, interior points, Hausdorff property.

Bounded sets, statements of I.u.b. axiom and its consequences, supremum and infimum, maximum and minimum, Archimedean property and its applications, density of rationals.

Unit II: Sequences (15 Lectures)
Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

Convergence of standard sequences like $\left(\frac{1}{1+na}\right)$ $\forall a > 0$, (b^n) $\forall b, 0 < b < 1$, $(c^{\frac{1}{n}})$ $\forall c > 0$, & $(n^{\frac{1}{n}})$
Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of $\left(\left(1+\frac{1}{n}\right)^n\right)$

Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse.

Unit III: Limits and Continuity (15 Lectures)

Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions, (when defined) Inverse of a bijective function.

Graphs of some standard functions such as $|x|, e^x, \log x, ax^2+bx+c, \frac{1}{x}, x^n n \geq 3), \sin x, \cos x, \tan x, \sin\left(\frac{1}{x}\right), x^2$ over suitable intervals of $\mathbb{R}$.

Definition of Limit $\lim\limits_{x \longrightarrow a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\epsilon - \delta$ definition, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit $\lim\limits_{x \longrightarrow a^-} f(x)$, right-hand-limit $\lim\limits_{x \longrightarrow a^+} f(x)$, non-existence of limits, $\lim\limits_{x \longrightarrow -\infty} f(x)$, $\lim\limits_{x \longrightarrow \infty} f(x)$ and $\lim\limits_{x \longrightarrow a} f(x) = \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

1. R.R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

3. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

Additional Reference Books

1. T.M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte, Ltd.

2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.

3. Ajit kumar- S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

4. James Stewart, Calculus, Third Edition, Brooks/ cole Publishing Company, 1994.

5. Ghorpade, Sudhir R.-Limaye, Balmohan V., A Course and Real Analysis, Springer International Ltd.2000.

ALGEBRA I
USMT 102

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgans laws, Cartesian product of two sets, Relations, Permutations ${}^nP_r$ and Combinations ${}^nC_r$.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I : Integers & Divisibility (15 Lectures)

Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle.

Divisibility in integers, division algorithm, greatest common divisor (GCD) and least common multiple (L.C.M) of two integers, basic properties of GCD such as existence and uniqueness of GCD of integers a & b and that the GCD. can be expressed as $ma + nb$ for some $m, n\mathbb{Z}$, Euclidean algorithm, Primes, Euclids lemma, Fundamental Theorem of arithmetic, The set of primes is infinite.

Congruence, definition and elementary properties, Eulers ϕ function, statements of Eulers theorem, Fermats theorem and Wilsons theorem, Applications.

Unit II : Functions and Equivalence relations (15 Lectures)

Definition of function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, Addition modulo$n$, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)

Definition of a polynomial, polynomials over the field F where $lF = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

Division algorithm in $\mathcal{F}[X]$ (without proof), and g.c.d of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.

A polynomial of degree over n has at most n roots roots, Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, A non-constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number $\frac{p}{q}$ to be a root a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is a irrational number where p is a prime number, roots of unity, sum of all the roots of unity.

Reference Books:

David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd. Norman L.

PRACTICALS FOR F.Y.B.Sc
USMTP01 Practicals

A. **Practicals for USMT101:**

(1) Application based examples of Archimedean property, intervals, nieghbourhood, interior point, Absolute Value

(2) Consequences of l.u.b axiom, infimum and supremum of sets Bounded sets

(3) Calculating limits of sequences, Cauchy sequences, monotone sequences.

(4) Limits of function and Sandwich theorem, continuous and discontinuous functions.

(5) Miscellaneous Theoretical Questions based on full paper.

B. **Practicals for USMT102:**

(1) Mathematical induction ,Division Algorithm and Euclidean algorithm in $\mathbb{Z}$, primes and the Fundamental theorem of Arithmetic.

(2) Congruence Euler's ϕfunction, Fermat's little theorem, Euler's theorem and Wilson's theorem.

(3) Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions. Equivalence relation.

(4) Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

(5) Miscellaneous Theoretical Questions based on full paper.

TUTORIALS FOR F.Y.B.A

Tutorials for UAMT101:

(1) Application based examples of Archimedean property, intervals, neighbourhood.

(2) Consequence of l.u.b axion, infimum and supremum of sets.

(3) Calculating limits of sequences.

(4) Cauchy sequences, monotone sequences.

(5) Limit of a function and Sandwich theorem.

(6) Continuous and discontinuous function.

(7) Miscellaneous Theoretical Questions based on full paper.

Semester II
USMT 201 CALCULUS II

UNIT - I: Series

Series $\sum_{n=1}^{\infty} a_n$ of real numbers, simple examples of series, Sequence of partial sums of a series, convergence of a series, convergent series, divergent series.

Necessary condition: $\sum_{n=1}^{\infty} a_n$ converges $\Longrightarrow a_n \longrightarrow 0$, but converse not true, algebra of convergent series, Cauchy Criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$, Comparison test, limit comparison test, alternating sereis, Leibnitz's theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely. Ratio test (without proof), root test (without proof) and examples.

UNIT - II: Continuity and differentiability of functions

Continuous function: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain,Sequential continuity, Algebra of continuous functions, Discontinuous functions ,examples of removable and essential discontinuity.

Intermediate Value theorem and its applications, Bolzano-Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds. Differentiation of real valued function of one variable: Definition of differentiation a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, algebra of differentiable functions.

Chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples)

UNIT - III: Applications of differentiation

Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, point of inflection.

Rolle's Theorem, Lagranges's and Cauchy's Mean Value Theorems, applications and examples, Monotone increasing and decreasing functions, examples.

L-Hospital rule without proof, examples of indeterminate forms, Taylor's theorem with Langrange's form of remainder with proof, Taylor polynomial and applications.

Reference books:

1. R.R.Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.

3. T.M.Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.

4. Ghorpade, Sudhir R, -Limaye, Balmohan V, A course in Calculus and Real Analysis, Springer International Ltd, 2000.

Additional Reference:

1. Richard Courant- Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.

2. Ajit Kumar- S.Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.

3. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.

4. G.B.Thomas, Calculus, 12th Edition 2009

ALGEBRA II

Prerequisites: Review of vectors in $\mathbb{R}^2, \mathbb{R}^3$ and as points, Addition and scalar multiplication of vectors in terms of co-ordinates, dot-product structure, Scalar triple product, Length (norm) of a vector.

Unit I System of Equations and Matrices (15 Lectures)

Parametric Equation of Lines and Planes , System of homogeneous and non-homogeneous linear Equations, The solution of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 2), (3, 3)$; Definition of n-tuple of real numbers, sum of n-tuples and scalar multiple of n-tuple.
Matrices with real entries; addition, scalar multiplication of matrices and multiplication of matrices, transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrices, symmetric matrix, skew symmetric matrix, invertible matrix; Identities such as $(AB)^t = A^tB^t, (AB)^{-1} = A^{-1}B^{-1}$.

System of linear equations in matrix form, Elementary row operations, row echelon matrix, Gaussian elimination method. Deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II Vector Spaces (15 Lectures)

Definition of real vector space, Examples such as $\mathbb{R}^n, \mathbb{R}[X], M_{m\times n}(\mathbb{R})$, space of real valued functions on a non-empty set.

Subspace: definition, examples: lines , planes passing through origin as subspaces of respectively; upper triangular matrices, diagonal matrices, symmetric matrices, skew symmetric matrix as subspaces of $M_n(\mathbb{R})(n = 2, 3); P_n(X) = a_o + a_1x + a_2x^2 + \cdots + a_nx^n, a_i \in \mathbb{R}, \forall 1 \leq i \leq n$ as subspace of $\mathbb{R}[X]$, the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbb{R}^n$.

Properties of a subspace such as necessary and sufficient conditions for a non-empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is the subset of other.

Finite linear combination of vectors in a vector space; linear span $L(S)$ of a non-empty subset S of a vector space, S is a generating set for $L(S), L(S)$ is a vector subspace of V; Linearly independent/ Linearly Dependent subsets of a vector space, a subset $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if and only $\exists i \in \{1, 2, \ldots k\}$ such that v_i is a linear combination of other vectors $v_j s$.

Unit-III Basis of a Vector Space and Linear Transformation (15 Lectures)

Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two basis of a vector space have same number of elements, any set of n linearly independent vectors in an n-dimensional vector space is a basis, any collection of $n+1$ vectors in an n-dimensional vector space is linearly dependent.

Extending any basis of a subspace W of a vector space V to a basis of the vector space V. If W_1, W_2 are two subspaces of a vector space V then $W_1 + W_2$ is a subspace of the vector space V of dimension $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.

Linear Transformations; Kernel, Image of a Linear Transformation T , Rank T , Nullity T, and properties such as: kernel T is a subspace of domain space of T and Img T is a subspace of co-domain space of T. If $V = \{v_1, v_2, \ldots, v_n\}$ is a basis of V and $W = \{w_1, w_2, \ldots, w_n\}$ any vectors in W then there exists a unique linear transformation $T : V \longrightarrow W$ such that $T(v_j) = w_j \ \forall j, 1 \leq j \leq n$, Rank nullity theorem (statement only) and examples.

Reference Books:

1. Serge Lang, Introduction to Linear Algebra, Second edition Springer.
2. S. Kumaresan , Linear Algebra , Prentice Hall of India Pvt limited .
3. K.Hoffmann and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi, 1971.
4. Gilbert Strang , Linear Algebra and its Applications, International Student Edition.
5. L. Smith , Linear Algebra, Springer Verlang.
6. A. Ramchandran Rao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.
7. T. Banchoff and J. Warmers: Linear Algebra through Geometry, Springer Verlag, New York, 1984.
8. Sheldon Axler: Linear Algebra done right, Springer Verlag, New York.
9. Klaus Janich': Linear Algebra.
10. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
11. Gareth Williams: Linear Algebra with Applications.

PRACTICALS FOR F.Y.B.Sc
USMTP02-Practicals

A. **Practicals for USMT201:**

(1) Calculating limit of series, Convergence tests.

(2) Properties of continuous and differentiable functions.

(3) Higher order derivatives, Leibnitz theorem. Mean value theorems and its applications.

(4) Extreme values, increasing and decreasing functions. Applications of

(5) Taylors theorem and Taylors polynomials

(6) Miscellaneous Theoretical Questions based on full paper

B. **Practicals for USMT202:**

(1) Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1, 2), (1, 3), (2, 2), (2, 2), (3, 3)$, Row echelon form.

(2) Solving system $AX = b$ by Gauss elimination method, Solutions of system of Linear Equations.

(3) Examples of vector spaces , Subspaces,

(4) Linear span of an non-empty subset of a vector space, Basis and Dimension of Vector Space.

(5) Examples of Linear Transformation, Computing Kernel, Image of a linear map , Verifying Rank Nullity Theorem.

(6) Miscellaneous Theoretical Questions based on full paper.

TUTORIALS FOR F.Y.B.A

Tutorials for UAMT201:

(1) Calculating limit of series, Convergence tests.

(2) Properties of continuous functions.

(3) Differentiability, Higher order derivatives, Leibnitz theorem.

(4) Mean value theorems and its applications.

(5) Extreme values, increasing and decreasing functions.

(6) Applications of Taylors theorem and Taylor's polynomials.

(7) Miscellaneous Theoretical Questions based on full paper.

Scheme of Examination

I. **Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT101/UAMT101, USMT102 of Semester I and USMT201/UAMT201, USMT202 of semester II to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.

b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.

c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. **Semester End Examinations Practicals:**

At the end of the Semesters I & II Practical examinations of three hours duration and100 marks shall be conducted for the courses USMTP01, USMTP02.
In semester I, the Practical examinations for USMT101 and USMT102 are held together by the college.
In Semester II, the Practical examinations for USMT201 and USMT202 are held together by the college.

Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	Marks out of	duration
USMTP01	Questions from USMT101	Questions from USMT102	80	3 hours
USMTP02	Questions from USMT201	Questions from USMT202	80	3 hours

Marks for Journals and Viva:
For each course USMT101/UAMT101, USMT102, USMT201:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester I and II shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.